An Application of Assignment Problem in Agriculture Using R

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Authors’ contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JSRR/2017/31902

Editor(s):
(1) Narayan Thapa, Department of Mathematics and Computer Science, Minot State University, 58707 Minot, ND, USA.

Reviews:
(1) Abdullah Sonmezoglu, Bozok University, Turkey.
(2) William P. Fox, NPS, USA.

Complete Peer review History: http://www.sciencedomain.org/review-history/18117

Received 29\textsuperscript{th} January 2017
Accepted 11\textsuperscript{th} February 2017
Published 9\textsuperscript{th} March 2017

ABSTRACT

In practical situations, sometimes we are faced with different type of problems which consists of relationships such as labors to fields, fertilizers to fields, jobs to machines, men to offices etc. where assignees possess varying degree of efficiency, called as cost or effectiveness. Such types of problems are solved using assignment techniques. There are different methods for solving the assignment problems but among these the Hungarian method provides us an efficient procedure for finding out optimal solution of an assignment problem. This paper is concerned with the special class of allocation problems, where the objective is to find optimal assignment of the number of paddocks to the number of crops used. The optimal allocation is found using R-software.

Keywords: Linear programming; integer programming; assignment problem; optimal cost; Hungarian method; R-software.

1. INTRODUCTION

Many problems arise in agriculture planning, science and engineering that are important for economic as well as social point of view. Every day there is a need of more production in order to meet the increasing demands of the population. These continuously challenges in
agriculture demands for the scientific programming methods to identify optimal allocation of resources to meet the specification limits. Such problems are usually solved by optimization techniques like Assignment technique. The assignment problem involves assignment of jobs to workers on one-to-one pattern. The number of jobs is presumed to be equal to the number of workers. However, if this is not the case, either fictitious jobs or workers as required can be created to satisfy this assumption. The assignment problem involves the optimal allocation of various resources having varying degree of efficiencies to various jobs that are to be completed. Several authors have developed procedures for assignment problems, which consider only one-objective function. [1] Studied an algebraic approach to assignment problems. [2] Provide a solution for an assignment problem with multiple objectives. [3] developed some work on nonlinear assignment problems. [4] considered a special class of allocation problems to find the optimal assignment. [5] contributed interval Hungarian method that consider the concept of interval analysis for solving interval linear assignment problems. [6] developed a step-wise procedure using Solver tool in MS-Excel to solve an Assignment Problem by means of Linear Programming Technique.

In this paper, the special class of allocation problem is formulated and the optimal allocation is obtained using R-software. R software is similar to S-PLUS software and both are implementations of S language, developed at Bell laboratories USA, which is birth place of C language and UNIX operating system. Two fundamental books written by [7] and [8] are of immense use in understanding this software. [9] discuss in detail the application of R-software in agricultural data analyses. One of the important feature of R-software is that it is an open source and freely available software on website http://cran-project.org. R language is essentially a functional language for all practical purposes of data analysis and graphics.

Suppose there are \( n \) jobs which is to be performed and \( n \) persons are available for doing these jobs. Assume that each person can do each job at a time, though with different efficiency. Let \( c_{ij} \) be the cost if the \( i^{th} \) person is assigned to the \( j^{th} \) job. The problem is to find an assignment (which job should be assigned to which person on one – one basis) so that total cost of performing all jobs is minimized. Problems of this kind are known as assignment problem.

### 1.1 Generalized Format of Assignment Problem

The assignment problem can be mathematically written as

\[
\text{Find } X_{ij} > 0 \ (i,j = 1, \ldots, n) \text{ so as to minimize } \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \text{ subject to restrictions:}
\]

\[
\sum_{j=1}^{n} X_{ij} = 1 \ (\text{one job is done by the } i^{th} \text{ person})
\]

\[
\sum_{i=1}^{n} X_{ij} = 1 \ (\text{only one person should be assigned } j^{th} \text{ job})
\]

where \( z \) = total cost.

\[
X_{ij} = \begin{cases} 1 & \text{if the } i^{th} \text{ person is assigned } j^{th} \text{ job} \\ 0 & \text{if the } i^{th} \text{ person is not assigned } j^{th} \text{ job} \end{cases}
\]

We given some definitions for dealing with the formulations:

**Balanced assignment problem:** This is an assignment where the number of persons is equal to the number of jobs.

**Unbalanced assignment problem:** This is the case of assignment problem where the number of persons is not equal to the number of jobs. A dummy variable, either for a person or job (as it required) is introduced with zero cost or time to make it a balanced one.
Dummy job / person: Dummy job or person is an imaginary job or person with zero cost or time introduced in the unbalanced assignment problem to make it balanced one.

An infeasible Assignment: Infeasible assignment occurs when a person is incapable of doing certain job or a specific job cannot be performed on a particular machine. These restrictions should be taken in to account when finding the solutions for the assignment problem to avoid infeasible assignment.

2. THE HUNGARIAN METHOD: ALGORITHM

Step 1. Subtract the smallest entry in each row from all the entries of its row.

Step 2. Subtract the smallest entry in each column from all the entries of its column.

Step 3. Draw lines through appropriate rows and columns so that all the zero entries of the cost matrix are covered and the minimum number of such lines is used.

Step 4. Test for Optimality: (i) If the minimum number of covering lines is $n$, an optimal assignment of zeros is possible and we are finished. (ii) If the minimum number of covering lines is less than $n$, an optimal assignment of zeros is not yet possible. In that case, proceed to Step 5.

Step 5. Determine the smallest entry not covered by any line. Subtract this entry from each uncovered row, and then add it to each covered column. Return to Step 3.

2.1 Remarks on the Hungarian Method

To solve an assignment problem in which the goal is to maximize the objective function, multiply the profits matrix by -1 and solve the problem as a minimization problem.

The Hungarian method may yield an incorrect solution if the problem is unbalanced. Thus, any assignment problem should be balanced before it is solved by the Hungarian method.

2.2 Numerical Illustration

Let us consider an assignment problem, where a farmer intends to plant four different crops in each of four equal sized paddocks. Rows representing four different like C1, C2, C3 and C4 and Columns representing the four equal sized paddocks like P1, P2, P3 and P4. The nutrient requirements required for different crops vary and the paddocks vary in soil fertility. Thus the cost of the fertilizers which must be applied depends on which crop is grown in which paddock. The fertilizer costs for each crops in each paddock is given in table below. The farmer’s objective is to find the optimal assignment of paddocks to crops in such a manner that the total fertilizer cost becomes minimized. This example is a variant of [4].

Initial table of fertilizer costs ($)

<table>
<thead>
<tr>
<th>Crops</th>
<th>Paddocks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P1</td>
</tr>
<tr>
<td>C1</td>
<td>300</td>
</tr>
<tr>
<td>C2</td>
<td>600</td>
</tr>
<tr>
<td>C3</td>
<td>500</td>
</tr>
<tr>
<td>C4</td>
<td>900</td>
</tr>
</tbody>
</table>

Note: mixed cropping of paddocks is not permitted, nor may the same crop be grown in two paddocks.

2.3 Solution Procedure of the Assignment Problem

The solution of the above Assignment problem is obtained by using R software.

> library(lpSolve)
> assign.costs<-matrix(c(300,600,500,900,400,900,100,800,200,400,700,900,200,300,200,500), 4, 4)
> assign.costs

Optimal solution is given below

<table>
<thead>
<tr>
<th>Assignment</th>
<th>Fertilizer cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assign paddock P1 to crop 1</td>
<td>300</td>
</tr>
<tr>
<td>Assign paddock P2 to crop 3</td>
<td>100</td>
</tr>
<tr>
<td>Assign paddock P3 to crop 2</td>
<td>400</td>
</tr>
<tr>
<td>Assign paddock P4 to crop 4</td>
<td>500</td>
</tr>
<tr>
<td>Total</td>
<td>= 1300</td>
</tr>
</tbody>
</table>

Therefore, the optimum minimized cost is $ 1300
2.4 When Problem is Unbalanced

When the number of persons is not equal to the number of jobs the assignment problem is said to be unbalanced. If the number of jobs is less than the number of persons, some of the persons cannot assigned any job. We introduce one or more dummy jobs of zero duration to make the assignment problem balanced. On the other hand, if the numbers of persons are less than number of jobs, we add one or more dummy persons with duration time zero to balance the assignment problem. Thus an assignment problem is said to be unbalanced if the cost matrix is not a square matrix.

If we consider previous example in terms of unbalanced assignment problem as

<table>
<thead>
<tr>
<th>Crops</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>300</td>
<td>400</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>C2</td>
<td>600</td>
<td>900</td>
<td>400</td>
<td>300</td>
</tr>
<tr>
<td>C3</td>
<td>500</td>
<td>100</td>
<td>700</td>
<td>200</td>
</tr>
</tbody>
</table>

Since the cost matrix is not a square we add a dummy row 4 with all elements 0.

<table>
<thead>
<tr>
<th>Crops</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>300</td>
<td>400</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>C2</td>
<td>600</td>
<td>900</td>
<td>400</td>
<td>300</td>
</tr>
<tr>
<td>C3</td>
<td>500</td>
<td>100</td>
<td>700</td>
<td>200</td>
</tr>
<tr>
<td>C4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The solution of the above Assignment problem is obtained by using R software. In R software install a package namely lpSolve. Then assign the values of the problem in R software as given below

```r
> library(lpSolve)
> assign.costs <- matrix(c(300,600,500,0,400,900,100,0,200,400,700,0,0,0,300,200,0), 4,4)
> assign.costs
> lp.assign (assign.costs)
> lp.assign (assign.costs)$solution
```

Optimal solution is given below

<table>
<thead>
<tr>
<th></th>
<th>Fertilizer cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assign paddock P1 to crop 4</td>
<td>0</td>
</tr>
<tr>
<td>Assign paddock P2 to crop 3</td>
<td>100</td>
</tr>
<tr>
<td>Assign paddock P3 to crop 1</td>
<td>200</td>
</tr>
<tr>
<td>Assign paddock P4 to crop 2</td>
<td>300</td>
</tr>
<tr>
<td>Total</td>
<td>600</td>
</tr>
</tbody>
</table>

Since crop 4 is dummy, paddock D has assigned no job.

Therefore, the optimum minimized cost is $600

3. CONCLUSION

The assignment problem involves the optimal allocation of various resources having varying degree of efficiencies to various jobs that are to be completed. This paper shows how one can solve the assignment problem using R software. This method is easy to apply can be utilized to all type of assignment problems.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES